

# Intermediate Scales in Supersymmetric GUTs: the Survival of the Fittest

Charanjit S. Aulakh<sup>(1)</sup>, Borut Bajc<sup>(2)</sup>, Alejandra Melfo<sup>(3)</sup>, Andrija Rašin<sup>(4,5)</sup>  
and Goran Senjanović<sup>(5)</sup>

<sup>(1)</sup> *Dept. of Physics, Panjab University, Chandigarh, India*

<sup>(2)</sup> *J. Stefan Institute, 1001 Ljubljana, Slovenia*

<sup>(3)</sup> *CAT, Facultad de Ciencias, Universidad de Los Andes, Mérida, Venezuela*

<sup>(4)</sup> *Dept. of Physics and Astronomy, University of North Carolina, Chapel Hill, NC 27599, U.S.A.*

<sup>(5)</sup> *International Center for Theoretical Physics, Trieste, Italy*

## Abstract

We show that intermediate scales in supersymmetric grand unified theories may exist naturally. Their origin is traced to the violation of the survival principle: in supersymmetry internal symmetries often forbid cubic couplings in the superpotential. This leads to a plethora of light supermultiplets whose masses are generated only by higher dimensional operators and thus suppressed by the cut-off scale. These new states often carry exotic quantum numbers and may be, in some cases, accessible to experiments in the near future.

## I. INTRODUCTION

Two main motivations for the low energy supersymmetry are the alleviation of the hierarchy problem and the successful unification of gauge couplings in the context of the minimal supersymmetric standard model (MSSM) [1]. The quest for supersymmetry has become one of the central issues of present day particle physics.

Today, on the other hand, we have growing observational evidence in favor of non-vanishing neutrino masses. The most natural and the simplest way to generate small neutrino masses is provided by the see-saw mechanism [2]. The accumulation of solar and atmospheric neutrino data points to the see-saw (right-handed neutrino) mass scale  $M_R$  in the range of  $10^{10} - 10^{15}$  GeV [3]. For believers in supersymmetry there is thus an important question as to whether there can be an intermediate scale below the grand unification scale  $M_X$ . The conventional wisdom in the context of minimal grand unified theories without extra fine-tuning seems to be no.

Recently, we have pointed out [4] an important connection between the renormalizable see-saw mechanism and low energy supersymmetry. First, if B–L symmetry is spontaneously broken, we can show that  $R$ -parity is an exact symmetry and the low-energy effective theory is the MSSM. Second, in many cases supersymmetry is consistent with the canonical form of the see-saw, an important achievement. It is then essential to know whether we can have  $M_R$  in the intermediate regime, as may be needed by neutrino data. In this letter we

wish to show that, contrary to the conventional wisdom, it may be quite natural to have  $M_R$  as an intermediate scale. In the minimal supersymmetric  $SO(10)$  theory  $M_R$  can be as low as  $10^{13}$  GeV with the prediction of lowering  $M_X$  close to  $10^{15}$  GeV, giving thus the old  $d = 6$  proton decay mode  $p \rightarrow \pi^0 e^+$  as the possibly dominant one. The reason for a different prediction from the minimal supersymmetric  $SU(5)$  stems from the violation of the survival principle [5] and it has generic features beyond any particular model. In other words, supersymmetric GUTs beyond  $SU(5)$  contain many potentially light supermultiplets which may modify the usual gauge coupling running. This is an important, albeit simple point, often overlooked in the literature (but not always, see for example [6,7]). Since it is generic to supersymmetric gauge theories, we address it more carefully and illustrate it in a number of examples, starting from a minimal  $U(1)$  model, all the way to the supersymmetric  $SO(10)$ . In all cases we will find a violation of the naive survival principle.

## II. SURVIVAL OF THE FITTEST

The survival principle is the natural assumption that a particle in multi scale theories takes the largest possible mass consistent with the symmetries of the theory. In particular, this means that the singlets under a particular gauge symmetry have masses that correspond to the scale of the larger symmetry under which they are not singlets. Of course, if they do not participate in symmetry breaking and have a gauge invariant mass, then they can go to an even higher scale.

An important aspect of this principle is the usual manifestation of the Higgs mechanism in ordinary gauge theories. When a multiplet  $\Phi$  develops a nonvanishing vacuum expectation value (VEV) through the potential

$$V = -m^2 \Phi^\dagger \Phi + \lambda^2 (\Phi^\dagger \Phi)^2 + \dots, \quad (1)$$

the Higgs scalars get masses of the order

$$m_\Phi \approx \lambda \langle \Phi \rangle \quad (2)$$

except for the unphysical Higgs and the possible pseudo-Goldstone bosons in the case of accidental symmetries of the potential. Of course, in general we have more self couplings in (1);  $\lambda$  stands generically for all of them. Notice that there is no symmetry that forbids the coupling  $\lambda$ , in as much as there is no symmetry that forbids the mass term. This is intimately related to the problem of hierarchies and thus we should not be surprised if the situation gets drastically different in supersymmetric theories.

In other words, unless one resorts to the unnatural fine-tuning  $\lambda \ll 1$ , we expect  $m_\Phi \sim \langle \Phi \rangle$ . This looks rather natural, so natural, that it is normally taken for granted even in supersymmetry. It is however potentially wrong as we show now. In supersymmetry the  $\Phi^4$  term in (1) arises from the superpotential  $W = \lambda \Phi^3/3 + \dots$ . However, the cubic term is often forbidden for symmetry reasons and so is the quartic self-coupling in the potential. In general there will be higher dimensional terms of the form

$$W_{nr} = \frac{\Phi^{n+3}}{M^n}, \quad (3)$$

where  $M$  is the cutoff scale of the theory;  $n > 0$  and it depends on the symmetry in question. Thus the  $\lambda$  coupling is strongly suppressed,  $\lambda_{\text{eff}} \sim \langle \Phi \rangle^n / M^n$ , and

$$m_\Phi \approx \lambda_{\text{eff}} \langle \Phi \rangle \approx \frac{\langle \Phi \rangle^{n+1}}{M^n} . \quad (4)$$

Even for  $n = 1$ , we can have  $m_\Phi \sim \langle \Phi \rangle^2 / M \ll \langle \Phi \rangle$ , since  $\langle \Phi \rangle \ll M$ . Here  $\Phi$  stands for the whole super-multiplet of bosons and fermions, since we will deal with  $\langle \Phi \rangle \gg M_W$  when supersymmetry is assumed to be a good symmetry. This fact is clearly important for phenomenology, but also for the unification of couplings. If  $M \sim M_X$ , and  $\langle \Phi \rangle = M_I$ , the intermediate scale, many particles will start contributing to running at much lower energies  $M_I^2 / M_X$ . In what follows we give some examples of the above with growing complexity and physical relevance.

### A. A U(1) prototype example

Take the simplest supersymmetric anomaly-free U(1) gauge symmetry with two chiral superfields  $\Phi$  and  $\bar{\Phi}$  (charges  $\pm 1$ ). The U(1) symmetry forbids cubic couplings and the renormalizable superpotential is trivial,  $W = m\Phi\bar{\Phi}$ . It implies no breaking of the symmetry,  $\langle \Phi \rangle = \langle \bar{\Phi} \rangle = 0$ . In order to have symmetry breaking, one needs at least a  $d = 4$  term in the superpotential,

$$W = m\Phi\bar{\Phi} + \frac{(\Phi\bar{\Phi})^2}{2M} , \quad (5)$$

which together with

$$V_D = \frac{g^2}{2} (|\Phi|^2 - |\bar{\Phi}|^2)^2 \quad (6)$$

provides nonvanishing VEVs:  $\langle \Phi \rangle = \langle \bar{\Phi} \rangle = \sqrt{mM}$ . Obviously, the effective coupling is suppressed

$$\lambda \sim \langle \Phi \rangle / M . \quad (7)$$

Whereas the combination  $\Phi - \bar{\Phi}$  belongs to the super-Higgs multiplet (mass of order  $g\langle \Phi \rangle$ ),  $\Phi + \bar{\Phi}$  has a smaller mass

$$m_{(\Phi+\bar{\Phi})} \approx \langle \Phi \rangle^2 / M . \quad (8)$$

This simple model portrays well the general situation we will find in the following.

### B. The supersymmetric Left-Right model

The minimal such model mimics completely the above U(1) example. The gauge group is  $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$  [8]. We have left handed  $(\Delta, \bar{\Delta})$  and right handed  $(\Delta_c, \bar{\Delta}_c)$  triplets and in the absence of nonrenormalizable terms there is no interaction in the superpotential [9]. Again, one needs nonrenormalizable terms to generate symmetry breaking.

Just as above, except for the super-Higgs multiplet, the rest of the particles should have masses of the order  $M_R^2/M$ , where  $M_R = \langle \Delta_c \rangle = \langle \overline{\Delta}_c \rangle$  is the symmetry breaking scale of parity and  $SU(2)_R$  and  $M$  is the cutoff.

The neutral multiplets in  $\Delta_c$  and  $\overline{\Delta}_c$  get a VEV, and the corresponding masses through the super-Higgs mechanism. The states that do not belong to the super-Higgs multiplet are one neutral complex combination and two doubly charged chiral multiplets from  $\Delta_c$  and  $\overline{\Delta}_c$ , and of course all the states from  $\Delta$  and  $\overline{\Delta}$ . This leads to an important prediction of light doubly charged supermultiplets potentially observable by experiment [10], with striking signatures at future colliders [11]. This result can be extended to include an arbitrary number of gauge singlet fields [12]. In this case the lightness of the doubly charged multiplets can be traced to their pseudo-Goldstone nature [12]. Namely, with parity odd singlet(s) one can generate the VEVs for the right handed fields, but the renormalizable superpotential has a larger accidental symmetry and the doubly charged fields are the Goldstone modes. They get masses if one includes the higher dimensional terms which break the accidental symmetry.

However, our point is much more general and is valid whenever one needs to invoke higher dimensional operators to generate relevant interactions. It says simply, as we already emphasized, that except for the super-Higgs mechanism, the rest of the supermultiplet masses are suppressed by the cut-off scale associated with the higher dimensional interactions. In the minimal model (without singlets) the lightness of the doubly charged states has nothing to do with the pseudo-Goldstone mechanism: simply, in the absence of higher dimensional operators there is no symmetry breaking whatsoever. When they are included and L-R symmetry is broken, the doubly charged states are light because they do not belong to the super-Higgs multiplet.

### III. SUPERSYMMETRIC $SO(10)$ THEORY

From the point of view of unification this will be the simplest illustration of the above idea. The minimal renormalizable theory requires the symmetric traceless **54** supermultiplet and the antisymmetric **45** adjoint. This allows for the breaking of  $SO(10)$  down to  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ . Further breaking is achieved by either (i) **126** and **126** or (ii) **16** and **16**. The former turns out to be more interesting so we study it first in detail.

#### A. **126** and **126** or renormalizable see-saw mechanism

A renormalizable  $SO(10)$  theory with a see-saw requires the minimum set of Higgs representations which break  $SO(10)$  down to the MSSM [13]

$$S = \mathbf{54}, \quad A = \mathbf{45}, \quad \Sigma = \mathbf{126}, \quad \overline{\Sigma} = \overline{\mathbf{126}}. \quad (9)$$

Although  $SO(10)$  is anomaly-free, as is well-known, one has to use both  $\Sigma$  and  $\overline{\Sigma}$  in order to ensure the flatness of the D-piece of the potential at large scales  $\gg M_W$ . The most general superpotential containing  $S, A, \Sigma$  and  $\overline{\Sigma}$  is

$$W = \frac{m_S}{2} \text{Tr } S^2 + \frac{\lambda_S}{3} \text{Tr } S^3 + \frac{m_A}{2} \text{Tr } A^2 + \lambda \text{Tr } A^2 S + m_\Sigma \Sigma \bar{\Sigma} + \eta_S \Sigma^2 S + \bar{\eta}_S \bar{\Sigma}^2 S + \eta_A \Sigma \bar{\Sigma} A . \quad (10)$$

We wish to obtain the pattern of symmetry breaking

$$\begin{aligned} SO(10) &\xrightarrow{\langle S \rangle = M_X} SU(2)_L \times SU(2)_R \times SU(4)_C \\ &\xrightarrow{\langle A \rangle = M_C} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \\ &\xrightarrow{\langle \Sigma \rangle = \langle \bar{\Sigma} \rangle = M_R} SU(2)_L \times U(1)_Y \times SU(3)_C \end{aligned} \quad (11)$$

and this is achieved with non-vanishing VEVs in the directions

$$s = \langle (1, 1, 1)_S \rangle \quad a = \langle (1, 1, 15)_A \rangle \quad b = \langle (1, 3, 1)_A \rangle \quad \sigma = \langle (1, 3, 10)_\Sigma \rangle \quad \bar{\sigma} = \langle (1, 3, \overline{10})_{\bar{\Sigma}} \rangle \quad (12)$$

(where here and in the following we label fields by their  $SU(2)_L \times SU(2)_R \times SU(4)_C$  quantum numbers). Notice that  $\langle S \rangle$  also preserves the discrete parity symmetry  $D_{LR}$  [14], which is then broken in the next step by the parity-odd singlet in  $(1, 1, 15)_A$ . This will of course cause the left-right inequality of masses already at the scale  $M_C$ .

The F-flatness equations are

$$\begin{aligned} m_S s + \frac{1}{2} \lambda_S s^2 + \frac{2}{5} \lambda (a^2 - b^2) &= 0 , \\ a(m_A + 2\lambda s) + \frac{1}{2} \eta_A \sigma \bar{\sigma} &= b(m_A - 3\lambda s) + \frac{1}{2} \eta_A \sigma \bar{\sigma} = 0 , \\ \sigma [m_\Sigma + \eta_A (3a + 2b)] &= 0 \end{aligned} \quad (13)$$

and  $\sigma = \bar{\sigma}$  guarantees D-flatness.

In order to get the required multi-scale symmetry breaking one needs  $s \gg a \gg \sigma = \bar{\sigma} \gg b \simeq \sigma^2/s$ . This can be achieved by paying the usual price of fine-tuning

$$m_A + 2\lambda s \simeq \frac{\sigma^2}{a} \ll s , \quad (14)$$

which then ensures

$$b \simeq \frac{\sigma^2}{s} \ll \sigma \quad (15)$$

(it is important to keep in mind that  $b$  can never vanish). A comment is in order. In the F flatness conditions (13) we ignore the fields in **16** and **10** dimensional representations. This is justifiable for the Standard Model non-singlet fields, but not for  $\tilde{\nu}^c$  in **16**. However, in the vacuum  $\sigma = \bar{\sigma}$ ,  $\langle \tilde{\nu}^c \rangle = 0$ , as can be easily proved following the similar analysis in the context of LR theories [9].

We now compute the spectrum of the theory. This has been discussed in reference [13], however only in the limit of a single step breaking of  $SO(10)$ . In this limit one misses the possibility of light states which have masses suppressed by the ratios of different scales of

symmetry breaking. In what follows we identify all such states together with the rest of the spectrum<sup>1</sup>.

At  $M_X$ ,  $(2, 2, 6)_S$  gets a mass of the order  $\langle S \rangle$  due to the super-Higgs mechanism. So do all the remaining fields in  $S$  and almost all of  $A$ , through the terms  $\text{Tr} S^3$  and  $\text{Tr} S A^2$ . The sole exception are the  $(1, 1, 15)$  fields in  $A$ , which we choose to have a much smaller mass imposing the fine-tuning condition (14): this multiplet will thus get a VEV in the next stage of symmetry breaking, at a much lower scale. Couplings with  $S$  also give a mass  $\sim M_X$  to all fields in  $\Sigma$  and  $\bar{\Sigma}$ , with the exception of the  $\text{SU}(4)$  decuplets. There is no fine-tuning involved here; this is automatic, since  $S$  is coupled only to  $\Sigma^2$  or  $\bar{\Sigma}^2$  and not to  $\Sigma\bar{\Sigma}$ .

In the next stage  $(1, 1, 15)_A$  will get a nonvanishing VEV in the direction of the  $\text{SU}(3)$  color singlet. Again, the super-Higgs mechanism gives a mass  $M_C$  to the colored triplets in  $(1, 1, 15)_A$ . The couplings with  $A$  provide masses at this scale to the left-handed components  $(3, 1, \bar{10})_\Sigma$  and  $(3, 1, 10)_{\bar{\Sigma}}$ , and to the colored fields in their right-handed analogues. As mentioned before the reason for the asymmetry in the left-right masses already at this scale comes from the fact that the singlet in  $(1, 1, 15)_A$  is parity-odd. But as we anticipated, the color octet and the singlet in  $A$  will survive the Pati-Salam breaking. The reason once again can be traced to the superpotential (10). Both  $A$  and the  $\Sigma$  ( $\bar{\Sigma}$ ) fields lack cubic self-interactions. In order to get effective interactions one has to integrate out the heavy field  $S$ . Notice that the color octets may get a contribution of the order  $M_R^2/M_C$ , which is just due to the necessary fine-tuning of formula (14).

At this point the situation resembles the Left-Right model discussed above. At the scale  $M_R$ , the  $\text{SU}(2)_R$  triplets in  $\Sigma$  and  $\bar{\Sigma}$  get their VEVs, but two doubly-charged chiral multiplet and one neutral combination remain light. The mass spectrum is summarized in the Table below.

State	Mass
all of $S$ in <b>54</b> all of $A$ in <b>45</b> , except $(1, 1, 15)_A$ all of $\Sigma$ in <b>126</b> + $\bar{\Sigma}$ in <b>126</b> , except $\text{SU}(4)_C$ decuplets	$\sim M_X$
$(3, 1, 10)_\Sigma + (3, 1, 10)_{\bar{\Sigma}}$ color triplets and sextets of $(1, 3, 10)_\Sigma$ and $(1, 3, \bar{10})_{\bar{\Sigma}}$ color triplets of $(1, 1, 15)_A$	$\sim M_C$
$(\delta_c^0 - \bar{\delta}_c^0), \delta_c^+, \bar{\delta}_c^+$ from the color singlets of $(1, 3, 10)_\Sigma$ and $(1, 3, \bar{10})_{\bar{\Sigma}}$	$\sim M_R$
color octet and singlet of $(1, 1, 15)_A$	$\sim \text{Max}[M_R^2/M_C, M_C^2/M_X]$
$(\delta_c^0 + \bar{\delta}_c^0), \delta_c^{++}, \bar{\delta}_c^{++}$ from the color singlets of $(1, 3, 10)_\Sigma$ and $(1, 3, \bar{10})_{\bar{\Sigma}}$	$\sim M_R^2/M_X$

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<sup>1</sup>For previous attempts in supersymmetric  $\text{SO}(10)$  theories with 126 fields see [15,16], where in [15] the presence of light states was noticed. However, the effects of  $M_I^2/M$  that are central to our paper were missed.

In addition, we have to include a **10** representation, which contains the MSSM Higgs doublets. Couplings with  $S$  (and the usual fine-tuning) will ensure that the colored fields in **10** get a mass  $\sim M_X$ . However, as is well known, at least two ten-dimensional Higgs representations are needed in general in order to generate non-vanishing quark mixing angles in the CKM matrix. This implies two extra Higgs doublets, which get a mass through the VEV  $b$  in  $A$  of the order  $M_R^2/M_X$ . The remaining fields in **10** and three generations of **16** (with the exception of the right-handed neutrinos) get their mass at  $M_W$ .

The Table clearly illustrates our point about the violation of the survival principle. The last two rows contain a number of states with masses below the corresponding scales of symmetry breaking. The other entries of the Table are clear and conform to the survival hypothesis. If one is weary of effective field theory arguments, it is a straightforward exercise to obtain the masses by direct calculation. For example the color octet states in  $(1, 1, 15)_A$  mix with their super heavy counterparts in  $(1, 1, 20)_S$ ; hence their see-saw-like suppressed mass.

The presence of these new states significantly alters the unification predictions. We wish to have a rough qualitative estimate of the new mass scales and thus it is only appropriate to perform this at the one-loop level. It is straightforward to write down the renormalization group equations and solve for  $M_X$ ,  $M_C$  and  $M_R$  in terms of the (unknown) unified coupling  $\alpha_U$ . We find that the most interesting scenario occurs for  $M_C^2/M_X > M_R^2/M_C$  (which fixes the mass of the octet).

We find, remarkably enough, that lowering  $M_R$  implies the lowering of  $M_X$ . This is an important result which changes the nature of the proton decay. Clearly, the proton decay constraints set a lower limit on  $M_X$ . At the same time we make sure that the value of  $\alpha_U^{-1}$  remains perturbative. With the scale of supersymmetry breaking  $\sim M_Z$ , requiring  $M_X \gtrsim 10^{15.5}$  GeV gives

$$M_C \gtrsim 10^{14.7} \text{ GeV} \quad , \quad M_R \gtrsim 10^{13.5} \text{ GeV} \quad , \quad (16)$$

for  $\alpha_3(M_Z) = 0.119$ . The lower limit on  $M_R$  (and  $M_C$ ) gets further decreased for higher  $\alpha_3(M_Z)$ , and for a typical MSSM value of  $\alpha_3(M_Z) = 0.126$  we get  $M_C \gtrsim 10^{14.3} \text{ GeV}$ ,  $M_R \gtrsim 10^{13.0} \text{ GeV}$ .

Notice that the scale  $M_C^2/M_X$  is actually bigger than  $M_R$ . In the Table we have shown it below only in order to separate new non-renormalizable mass scales from the original ones. The precise value of an intermediate scale  $M_R$  should be checked at the two-loop level including threshold effects (however see [17]), since it is not too far from the unification scale. Its existence is clearly important for neutrino masses through the see-saw, but its impact is even more dramatic on the proton decay predictions. If  $M_R$  lies below  $10^{14}$  GeV, the dominant proton decay mode may as well be into the positron and the neutral pion, as in the non-supersymmetric theories. This is in sharp contradiction with the minimal supersymmetric SU(5) theory in which this mode is highly suppressed and dominant modes would necessarily involve kaons, not pions (for a complete analysis and references see [18]). Of course, we may still have the kaon modes, however not necessarily dominant.

Anyway, the above example illustrates nicely our point: the combined effect of supersymmetry and internal symmetries may lead naturally to particles with masses much below the scale of symmetry breaking. Their impact on unification predictions may be non-trivial and should not be ignored. This is a dynamical question, though, that depends on the model

in question. In the next case it will turn out, in spite of the presence of potentially light states, that the theory chooses the minimal SU(5) route without any intermediate scale.

### B. 16 and $\overline{16}$ or large-scale R-parity breaking

Supersymmetric SO(10) models have been studied at length with the non-renormalizable version of the see-saw mechanism [19]. More precisely, one chooses one (or more) pair of “Higgs” in the spinorial representation **16** and  $\overline{16}$  whose VEVs induce B-L breaking and the mass for the right-handed neutrino through the  $d = 4$  terms:  $m_{\nu_R} \simeq \langle \mathbf{16}\overline{16} \rangle / M_{Pl}$ . This enables one to get rid of the large 126-dimensional representations which render the high-energy behavior above  $M_X$  non asymptotically free.

The disadvantage of this program, though, is that then R-parity is broken at a large scale  $M_R$ , and thus one needs additional, ad-hoc symmetries to understand the smallness of R-parity breaking at low energies. This appears to us more problematic than the loss of asymptotic freedom at super high energies. Even more important, it will turn out that, in spite of the possibly light particles, the unification constraints push all the scales towards  $M_X$ , leaving no room for intermediate scales. We still include this example in order to show the subtlety of the issue: it is a dynamical question whether or not one can have intermediate mass scales.

The superpotential closely resembles the previous case’s one

$$W = \frac{m_S}{2} \text{Tr } S^2 + \frac{\lambda_S}{3} \text{Tr } S^3 + \frac{m_A}{2} \text{Tr } A^2 + \lambda \text{Tr } A^2 S + (m_\psi + \eta_A A) \psi \overline{\psi} . \quad (17)$$

In complete analogy with case (i), the SO(10) symmetry forbids self-couplings for  $A$  and  $\psi$ . They will be generated as higher dimensional terms, i.e. when  $S$  is integrated out we get  $A^4/\langle S \rangle$ , and when  $A$  is integrated out one gets  $(\psi \overline{\psi})^2/\langle A \rangle$ . The pattern of symmetry breaking is taken again to be (11) with  $\psi$  in the role of  $\Sigma$ , and the mass spectrum is obtained following a similar reasoning. However, the LR breaking is now performed by SU(2)<sub>R</sub> *doublets*, and the only state in  $\psi, \overline{\psi}$  not participating in the super-Higgs mechanism is a neutral combination. The complete spectrum is now

State	Mass
all of $S$ in <b>54</b> all of $A$ in <b>45</b> , except $(1, 1, 15)_A$	$\sim M_X$
$(2, 1, 4)_\psi + (2, 1, \overline{4})_{\overline{\psi}}$ color triplets of $(1, 2, \overline{4})_\psi$ and $(1, 2, 4)_{\overline{\psi}}$ color triplets of $(1, 1, 15)_A$	$\sim M_C$
one neutral and two charged combinations from the color singlets of $(1, 2, \overline{4})_\psi$ and $(1, 2, 4)_{\overline{\psi}}$	$\sim M_R$
color octet and singlet of $(1, 1, 15)_A$	$\sim \text{Max} \left[ \frac{M_R^2}{M_C}, \frac{M_C^2}{M_X} \right]$
neutral combination from the color singlets of $(1, 2, \overline{4})_\psi$ and $(1, 2, 4)_{\overline{\psi}}$	$\sim \frac{M_R^2}{M_X}$



Only the color octet and the extra Higgs doublets could play some role in the running. Our calculations show that the only hope for unification is with  $M_C^2/M_X$  above  $M_R$ , but even in that case the only extra non-singlet fields below  $M_R$ , the heavy Higgs doublets, are not enough to change the usual MSSM picture, and no intermediate scales are present.

#### IV. CONCLUSIONS

The minimal supersymmetric standard model, if extrapolated to very high energies, predicts successfully the unification of gauge couplings. This would imply a desert above  $M_W$  all the way to the unification scale at  $10^{16}$  GeV, a rather dreadful scenario. Recently, there has been great excitement about the possibility of large compactified dimensions which would offer plenty of new physics to fill in the desert, even at scales as low as  $1 - 10$  TeV [20]. However, it is fair to say that the unification in this scenario [21] is still far from reaching the level of the usual supersymmetric field theory [1], and furthermore it is plagued by the severe proton decay problem.

Meanwhile, one is tempted to look for a possible oasis in the desert of energies between  $M_W$  and  $M_X$ . We have argued here that the supersymmetric SO(10) theory offers naturally such an oasis. It allows for a new scale associated with a see-saw mechanism below  $10^{14}$  GeV. Admittedly, this scale is too large to be of direct experimental interest, but it could play an important role for neutrino physics. Furthermore, it has a strong effect on proton decay suggesting a possibly dominant mode:  $p \rightarrow \pi^0 e^+$ . At the same time, in this theory R-parity is exact which solves the d=4 proton decay problem of low energy supersymmetry and guarantees the stability of the LSP (the lightest supersymmetric partner) [4]. This and other aspects of the theory will be discussed in a forthcoming publication.

The main reason behind the existence of a new scale below  $M_X$  is the violation of the survival principle as we have discussed at length. In many cases supersymmetric theories are characterized by the absence of cubic couplings in the superpotential; this leads to an important possibility of light states with masses of the order  $M_I^2/M$ , where  $M_I$  is the scale of relevant symmetry and  $M$  is the cut-off scale, as explained in the main body of this work. Such states, which often carry exotic quantum numbers, thus may lie at a scale much lower than  $M_I$  and possibly as low as the scales soon to be probed in new experiments. Even if the reader is not excited by the examples we have provided, we hope that she or he will find even more interesting theories which will not suffer from the monotony of the desert and which will offer new physics much closer to the electro-weak scale.

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